

Mathematical modeling of forest land quality monitoring

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Abstract Scale representation is an operation known especially, in the field of cartography, where the main objective is to draw the map with a certain degree of detail. On such a scale, relief details, for forestlands cannot be represented, as graphics resolution of the map is too small. Given the density of species and relief variety, the new map permits now representation of any trees species and any relief form, watercourses or forests. An easy to use and accurate map should prove a good compromise in scale of graphic representation and resolution. Resolution of representation in the mapping of forestlands increases with the scale of representation only effect of scaling is to increase and not to decrease the data imaging.

Key words

multiresolution, wavelet, signal, cartography

Correlation that exists between the scale and resolution representation of a signal is surprised in this paper as “multi-resolution theory”, leading to a very interesting approach for practical applications. It was materialized as a series of analysis-synthesis algorithms based on the multi-resolution structure of the usual signal space (Prueitt, 1995). To facilitate understanding of the construction proposed by Mallat (Mallat, 2008) and its generalizations we will present briefly some mathematical concepts related to orthogonality and projection that confers possibility to

design different signals on some Hilbert subspaces (Hilbert, 2004) with role in deparasiting unwanted perturbations signals as shown below (fig 1). Such a signal therefore carries not only useful information but also a parasite. Given the density of species and relief variety, the new map permits now representation of any trees species and any relief form, watercourses or forests (Bândiu, 1999). The mathematical model presented below identifies both surface acres of forest and all the properties required for evaluation.



Fig1 Satellite picture of a forestland (with parasites)

Removing the cloud formations data on the amount of timber on a surface can be identified and data on existing species on that surface. The mathematical model by identifying the wave function and insertion of requirements on quantitative and qualitative assessment of forest land allow optimization and restoring of information with high accuracy (Stanomir and Stanasila, 1981). Not all wavelets are suitable for a particular application.

Different applications need different wavelets with specified properties for effective and efficient processing. In the construction of a wavelet, some parameters can be chosen to model the wavelet to achieve certain purposes in its processing.

This paper emphasizes the procedure of wavelets construction starting from an orthonormalized *quadrature mirror filter* (QMF) (Jones, 2009).

In this sub-paragraph an orthonormalized filter QMF is noted $(\mathbf{a}_k, \mathbf{b}_k)$ where $\{\mathbf{a}_k\}$ is a low-pass filter and $\{\mathbf{b}_k\}$ is a high-pass filter. Supposing that filter length $L > 0$ is given by perfect reconstruction condition, which

is $\sum_{l=0}^{L-1} a_l a_{l+2k} = \delta_{ok}$ than normalization condition is

given by $\sum_{l=0}^{L-1} a_l = \sqrt{2}$. Combining both conditions an

equation system is $(\mathbf{a}_0, \mathbf{a}_1, \dots, \mathbf{a}_{L-1})$ and in the system there are L are unknown variables and

$(\frac{L}{2} + 1)$ possible equations. This means that

$\frac{L}{2} - 1$ free parameters can be chosen in the

modeling process of wavelets. Ingrid Daubechies (Daubechies, 1992) gives a method for constructing an orthogonal wavelet filter with compact support $\{H < G\}$ from QMF filters, where H is “low-pass filter” and $(\mathbf{b}_0, \mathbf{b}_1, \dots, \mathbf{b}_{L-1})$ is “high-pass filter”

determined of $\mathbf{b}_i = (-1)^i \mathbf{a}_{L-1-i}$. Since then,

several approaches of optimal wavelets modeling were proposed, that relaxed the convergence conditions for modeling based on other new criteria. A new class of wavelets was modeled with interesting approximation properties as wavelets best to represent an. For example, Wavelet model “*adapted to signal*” was formulated as *semi-infinite linear programming* (SIP).

In optimization theory, semi-infinite programming (SIP) is an optimization problem with a finite number of variables and an infinite number of constraints, or an infinite number of variables and a finite number of constraints. In the former case the constraints are typically parameterized (Bonnans *et al.* 2000).

Parameterizing FIR filters satisfy the condition of orthogonalization and normalizing and is given by

$$A_L(z) = \frac{1}{\sqrt{2}} \sum_{k=0}^{L-1} a_k z^k \text{ and}$$

$$B_L(z) = \frac{1}{\sqrt{2}} \sum_{k=0}^{L-1} b_k z^k .$$

By induction

$$\begin{pmatrix} A_L(z) \\ B_L(z) \end{pmatrix} = \begin{pmatrix} \frac{P_L(z)}{2} \\ (-1)^{\frac{L}{2}} \frac{Q_L(z)}{2} \end{pmatrix}$$

can be obtained and $P_i(z), Q_i(z)$ are determined by the following equations

$$\begin{pmatrix} P_i(z) \\ Q_i(z) \end{pmatrix} = \begin{pmatrix} \cos(\alpha_i - 1) & \sin(\alpha_i - 1) \\ -\sin(\alpha_i - 1) & \cos(\alpha_i - 1) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & z^2 \end{pmatrix} \begin{pmatrix} P_{i-1}(z) \\ Q_{i-1}(z) \end{pmatrix}$$

$$\text{and } \begin{pmatrix} P_1(z) \\ Q_1(z) \end{pmatrix} = \begin{pmatrix} \cos(\alpha_0) & \sin(\alpha_0) \\ -\sin(\alpha_0) & \cos(\alpha_0) \end{pmatrix} \begin{pmatrix} 1 \\ z \end{pmatrix}$$

, where $(\alpha_0, \alpha_1, \dots, \alpha_{\frac{L}{2}-1})$ are

constrained by $\sum_{j=0}^{\frac{1}{2}-1} = \pi / 4$. Parametric form of

the orthonormal filter QMF for $L = 4$ is represented here as an example.

$$A(\alpha_0, \alpha_1) = \{ \cos(\alpha_0)\cos(\alpha_1), \cos(\alpha_1)\sin(\alpha_0), \\ -\sin(\alpha_0)\sin(\alpha_1), \cos(\alpha_0)\sin(\alpha_1) \}$$

It can be noted that not all orthonormal filters QMF generate orthonormal bases in $L^2(\mathbb{R})$.

Due to non convergence, Lawton proposed a condition necessary and sufficient for the orthonormal filter to determine orthonormal wavelets (Lawton, 1991).



Fig 2 Cleansed (deparasitized) satellite image of a forestland

A simple approach for the wavelet orthonormal model based on particular criteria is outlined, as follows:

1. One establishes a criterion for optimal wavelet specific requirements based on the application.
2. One generate parameterization for orthonormal QMF filter
3. One optimize the parameters to determine the best filter
4. One checks if the filter optimized check condition of Lawton.

Optimizing satellite images using mathematical model of signal reconstruction can be done by eliminating the disturbing factors and by identifying suitable wavelet.

Conclusions

Images is the main type of data in areas such as satellite remote sensing, meteorology, cartography, forest management plans, but has a drawback because it occupies a large amount and transmission or archiving data is made in compressed format. There are specific methods of compression / decompression with or without loss of information. In compression with loss original reports are great so the image will be distorted and in for this purpose a large role is played by the multiresolution analysis (MRA) or multiscale approximation (MSA) of wavelet transforms presented above on a specific model.

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